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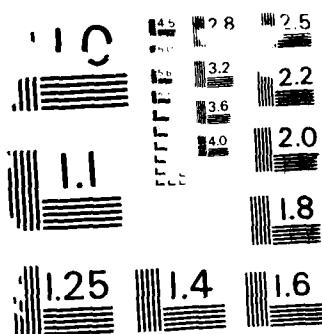
ELASTIC WAVE DISPERSION IN LAMINATED COMPOSITE PLATE
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INTRODUCTION

In the past dynamic behavior of infinite periodically laminated medium has been studied extensively. A review of the literature on exact and approximate analyses of this problem can be found in [1,2].

In this paper we use the stiffness method that was presented in [2] to study dispersion of waves in a laminated plate. In this approach each lamina is divided into several sublayers and the displacement distribution through the thickness of each sublayer is approximated by polynomial interpolation functions in such a way that displacements and tractions are continuous across the interfaces between adjacent sublayers. Details of the method can be found in [2,3]. Here we summarize the pertinent equations and discuss numerical results obtained for particular systems.

This study is motivated by a desire to model wave propagation in a continuous fiber-reinforced laminated plate. If the wavelength is long compared to the fiber diameters and spacing, then each lamina can be modeled as a homogeneous transversely isotropic medium with the symmetry axis parallel to the fibers. The overall effective elastic properties of such a medium can be calculated from the fiber and matrix properties by using an effective modulus theory [4,5]. Such an assumption has been made in this paper. Thus the dispersion characteristics of guided waves in a layered anisotropic plate has been studied here.

GOVERNING EQUATIONS

We consider time harmonic waves in a plate composed of m laminae. For simplicity in analysis it will be assumed that each lamina is transversely isotropic with the symmetry axis aligned with either the x - or the y -axis (Fig. 1). This assumption is not necessary for

the development of the equations, but it is made here to keep the algebra as simple as possible for the anisotropic problem at hand. Under this assumption the wave propagation problem reduces to two uncoupled ones: plane strain in which the displacement components are u_x , 0, u_z , and SH or antiplane strain when the only nonzero displacement component is u_y . So in the following we shall treat these two separately.

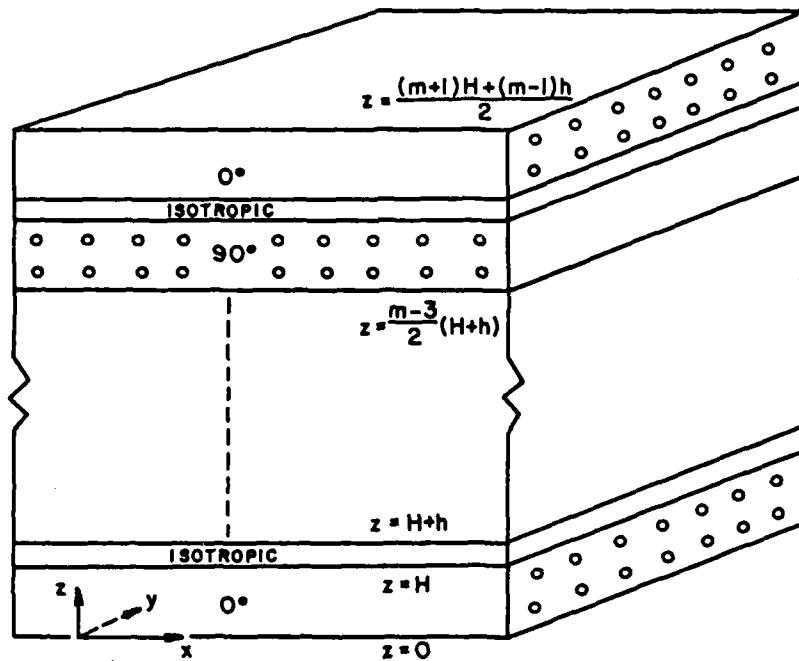


Fig. 1. Geometry of the laminated plate.

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Plane Strain

Consider the i th lamina bounded by $z=z_{i-1}$ and $z=z_{i+1}$. The stress strain relation in this lamina will be given by

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{zz} \\ \sigma_{xz} \end{Bmatrix} = \begin{Bmatrix} c_{11}^{(i)} & c_{13}^{(i)} & 0 \\ c_{13}^{(i)} & c_{33}^{(i)} & 0 \\ 0 & 0 & c_{66}^{(i)} \end{Bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{zz} \\ \gamma_{xz} \end{Bmatrix} \quad (1)$$

where σ_{ij} and ϵ_{ij} are the stress and strain components, respectively, and we have written $\gamma_{xz}=2\epsilon_{xz}$. Note that if y -axis is the axis of symmetry, then $c_{11}^{(i)}=c_{33}^{(i)}$ and $c_{66}^{(i)}=\frac{1}{2}(c_{11}^{(i)}-c_{13}^{(i)})$.

The problem is then equivalent to that of an isotropic one.

In order to get good numerical results each lamina will be divided into several sub-layers, M_i , say. Within the j th sublayer we will choose a local coordinate with the origin at the mid-plane and x_j , y_j , z_j , parallel to the global x , y , z axes, respectively. Let $2h_j$ be the thickness of this sublayer. Denoting $\tilde{u}_x^{(j)}$ to be the displacement at a point in the j th lamina we write

$$u_x^{(j)} = u_{j-1} f_1 + u_j f_2 + \left(\frac{1}{c_{66}^{(j)}} x_{j-1} - \frac{\partial \omega_{j-1}}{\partial x_j} \right) f_3 + \left(\frac{1}{c_{66}^{(j)}} x_j - \frac{\partial \omega_j}{\partial x_j} \right) f_4 \quad (2)$$

$$u_s^{(j)} = \omega_{j-1}f_1 + \omega_j f_2 + \left(\frac{1}{c_{ss}^{(j)}} \sigma_{j-1} - \frac{c_{13}^{(j)}}{c_{ss}^{(j)}} \frac{\partial u_{j-1}}{\partial x_j} \right) f_3 + \left(\frac{1}{c_{ss}^{(j)}} \sigma_j - \frac{c_{13}^{(j)}}{c_{ss}^{(j)}} \frac{\partial u_j}{\partial x_j} \right) f_4 \quad (2)$$

where f_n ($n=1, \dots, 4$) are cubic polynomials in the local coordinate z_j given by

$$f_1 = \frac{1}{4}(2-3\eta_j + \eta_j^3), \quad f_2 = \frac{1}{4}(z+3\eta_j - \eta_j^3) \quad (3)$$

$$f_3 = \frac{h_j}{4}(1-\eta_j - \eta_j^2 + \eta_j^3), \quad f_4 = \frac{h_j}{4}(-1-\eta_j + \eta_j^2 + \eta_j^3)$$

Here $\eta_j = z_j/h_j$ and $u_j, \omega_j, x_j, \sigma_j$ are the values of u_x, u_s, σ_{xx} , and σ_{ss} at the j th node. These nodal values of the displacement and traction components are functions of x_j ($= x$) and t . In this paper it will be assumed that the time dependence is of the form $e^{-i\omega t}$, ω being the circular frequency. The factor $e^{-i\omega t}$ will be dropped in the sequel.

The equations governing the nodal generalized coordinates $\{u_j, x_j, \omega_j, \sigma_j\}$ are obtained using Hamilton's principle. For details see [3]. It is found that the amplitudes of the generalized nodal variables satisfy the matrix equation

$$(k^4[\mathbf{K}_1] - ik^3[\mathbf{K}_2] - k^2[\mathbf{K}_3] + ik[\mathbf{K}_4] + [\mathbf{K}_5]) \{Q\} = 0 \quad (4)$$

where we have written

$$\{Q\} = \{\mathbf{Q}_0\} e^{ikx} \quad (5)$$

$\{Q\}$ being the vector of all the generalized coordinates. Matrices $[\mathbf{K}_1]$, etc., have been defined in [3]. Note that $[\mathbf{K}_3], \dots, [\mathbf{K}_5]$ depend linearly on ω^2 .

For nontrivial solution the determinant of the coefficient matrix of $\{Q_0\}$ must be zero. This is the dispersion equation to solve for k for given ω .

Antiplane Strain

The derivation given above is for the plane strain. The antiplane strain case can be considered in a similar manner and that results in a much simpler frequency/wave number equation. In this case we assume

$$u_y^{(j)} = v_{j-1}f_1 + v_j f_2 + \frac{1}{c_{ss}^{(j)}} r_{j-1} f_3 + \frac{1}{c_{ss}^{(j)}} r_j f_4 \quad (6)$$

Here $r = c_{13}^{(j)} \partial u_y^{(j)} / \partial z$. The corresponding eigenvalue problem for the wave number is

$$(-k^2[\mathbf{K}_6] + [\mathbf{K}_8]) \{Q\} = 0 \quad (7)$$

As illustrative examples we solved equations (4) and (7) for a fiber-reinforced (0°) homogeneous plate and a three-layered ($0^\circ/90^\circ/0^\circ$) plate with or without interface layers. These results are discussed in the next section.

NUMERICAL RESULTS AND DISCUSSION

As an application of the technique described above, we considered a fiber-reinforced plate when the fibers are aligned along the x -axis (0°). The properties of the plate are given in Table 1. As mentioned before, for propagation in the 0° (90°) direction the problem is tractable analytically. Figure 2 shows the frequency spectrum for propagation in the x -direction. Also shown in this figure is the comparison with the exact solution. It is

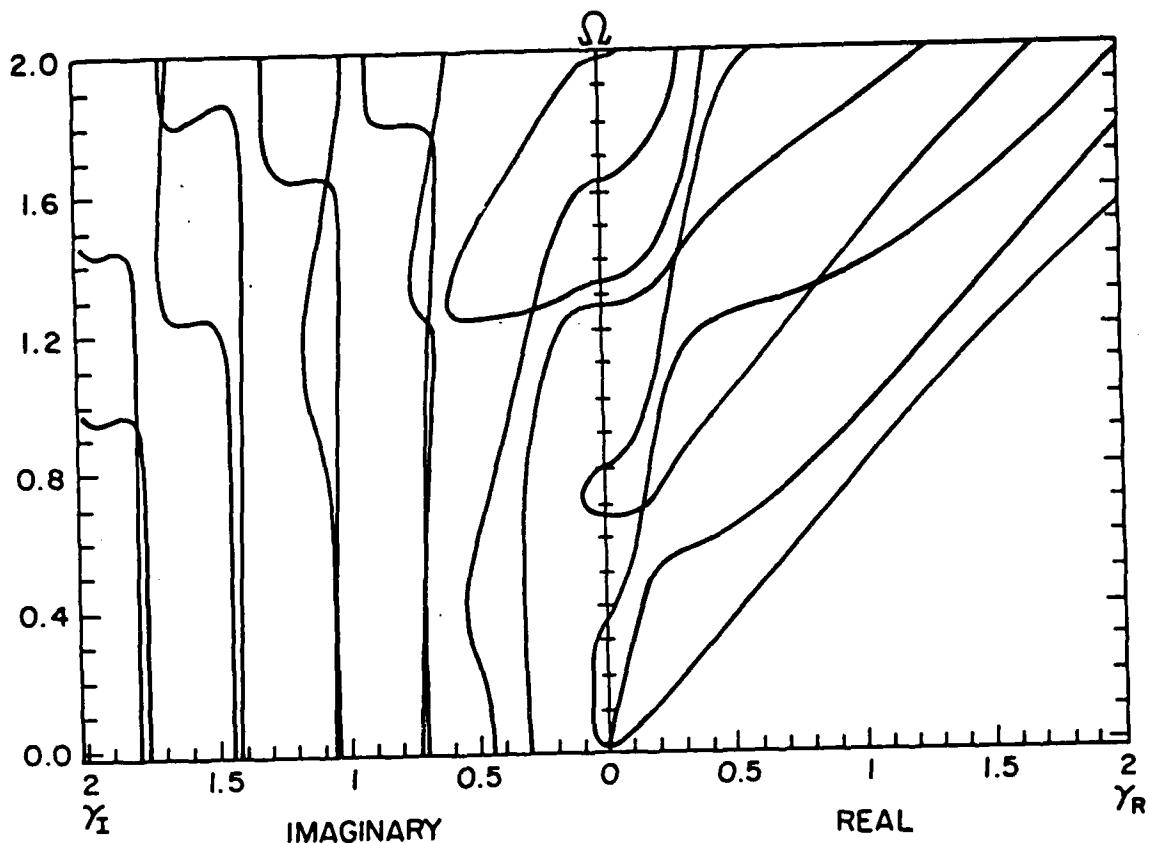


Fig. 4. Dispersion curves for a $0^\circ/90^\circ/0^\circ$ plate with soft interface thin layers.

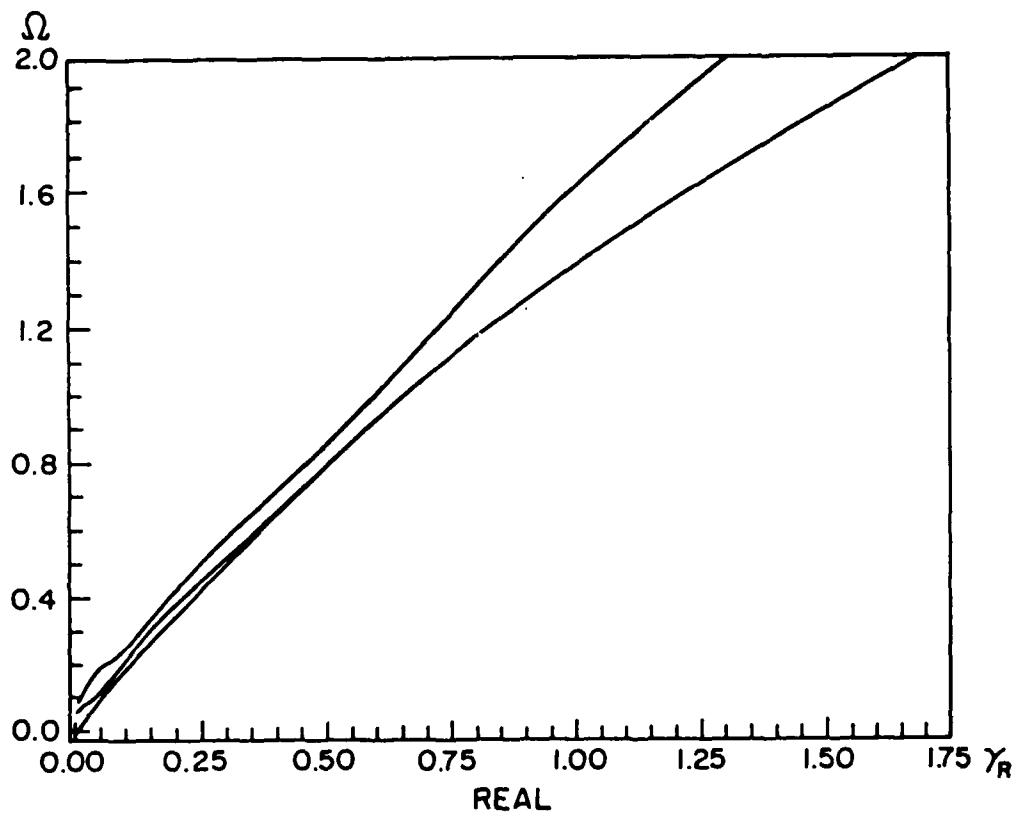


Fig. 5. First three real branches of the frequency spectrum for a $0^\circ/90^\circ/0^\circ$ plate with soft interface thin layers.

Finally, Fig. 6 shows the same effect on the first three branches of the frequency spectrum of SH waves.

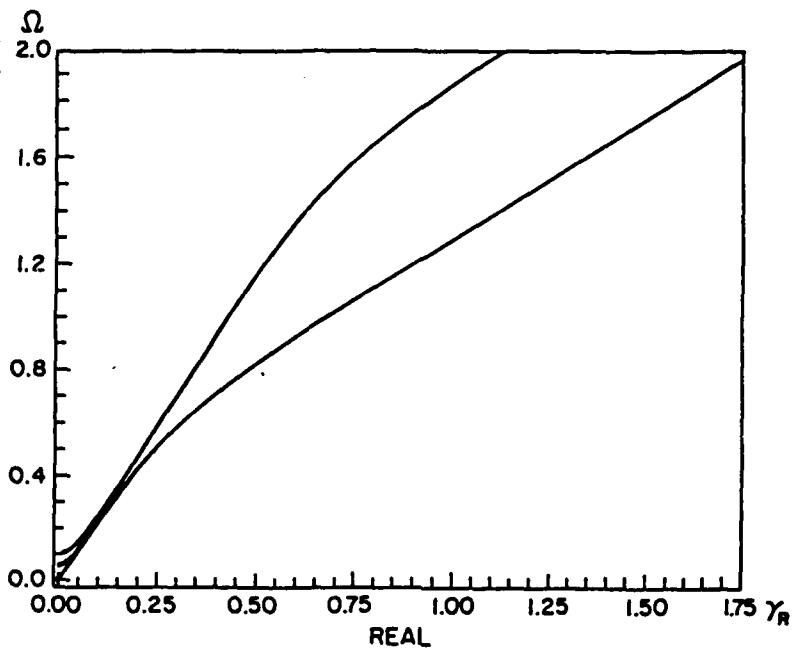


Fig. 6. First three real branches of the frequency spectrum of SH waves for a $0^\circ/90^\circ/0^\circ$ plate with soft interface thin layers.

CONCLUSION

Dispersion curves are presented for propagation of in-plane and out-of-plane (SH) waves propagating along the fibers in a fiber-reinforced plate. These curves differ significantly from those in an isotropic plate. It is also shown that the dispersion curves for waves propagating in a cross-ply ($0^\circ/90^\circ/0^\circ$) laminate with interface layers differ significantly from those for a laminate without interface layers and both of these are different than the ones for a single-layered fiber-reinforced plate. The most significant features of the curves for a plate with interface layers are the lowering of the cut-off frequencies and their slopes. This should have significant effect on the overall plate response and would allow ultrasonic characterization of interface properties.

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seen that the comparison is excellent. In this figure $\Omega = \frac{\omega H}{2\pi\sqrt{(c_{55}/\rho)\sigma}}$ and $\gamma = \frac{kH}{2\pi}$. It

is seen that as $k \rightarrow \infty$ the slopes of the first symmetric and antisymmetric modes tend to the ratio $V_R/\sqrt{C_{55}/\rho}$, where V_R is the Rayleigh wave velocity in the x-direction. For the stiffness method we used 15 sublayers.

Table 1. Properties of 0° and 90° laminae, and the interface layer. All the stiffness are in the units of 10^{11} N/m^2 .

	$\rho(\text{g/cm}^3)$	c_{11}	c_{22}	c_{12}	c_{44}	c_{55}
0° lamina	1.2	1.6073	0.1392	0.0644	0.0350	0.0707
Interface	1.8	0.0865	0.0865	0.0475	0.0195	0.0195
90° lamina	1.2	0.1392	1.6073	0.0644	0.0707	0.0350

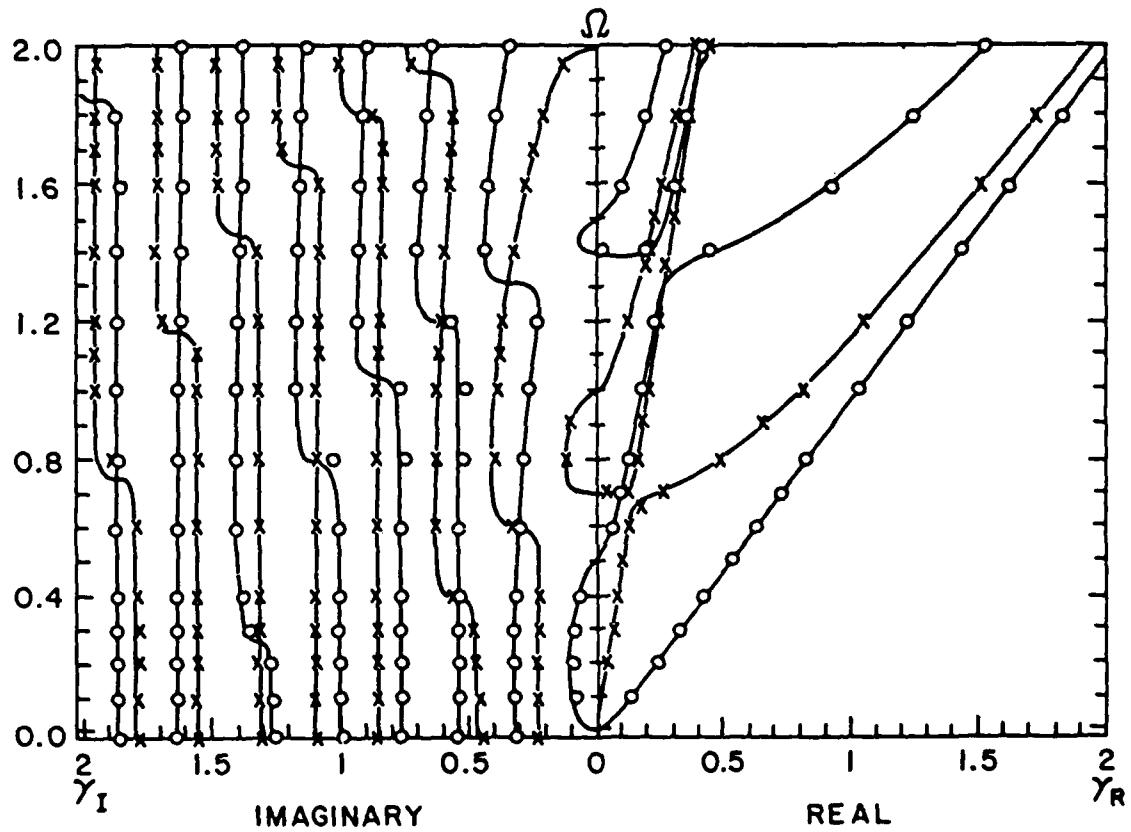


Fig. 2. Dispersion curves for the real and imaginary wave numbers. Solid lines are obtained by the present method. Symbols x (symmetric) and o (antisymmetric) are the exact calculations.

The next problem considered as a sandwich plate with $0^\circ/90^\circ/0^\circ$ configuration. Figure 3 shows the dispersion curves. These differ appreciably from the ones shown in Fig. 2. It is seen that the cut-off frequencies are lowered as well as the slopes of the curves. Branches of nonpropagating modes are also quite different.

Figure 4 shows the dispersion curves for a $0^\circ/90^\circ/0^\circ$ plate with two interface layers between the two outer laminae (0°) and one inner lamina (90°). It is assumed that thickness ratio of each interface layer and each adjacent lamina is 0.1. Cut-off frequencies are seen to be lower than those in Fig. 3. To see the effect of interface layers on the first three branches of the frequency spectrum we plotted renormalized Ω vs. γ in Fig. 5. In this figure $\Omega = \frac{\omega h}{\pi \sqrt{(c_{55}/\rho)_{\text{isotropic}}}}$ and $\gamma = \frac{kh}{\pi}$. It is interesting to note the slowing down of the waves as the wavelength becomes of the order of the thickness of the interface layers.

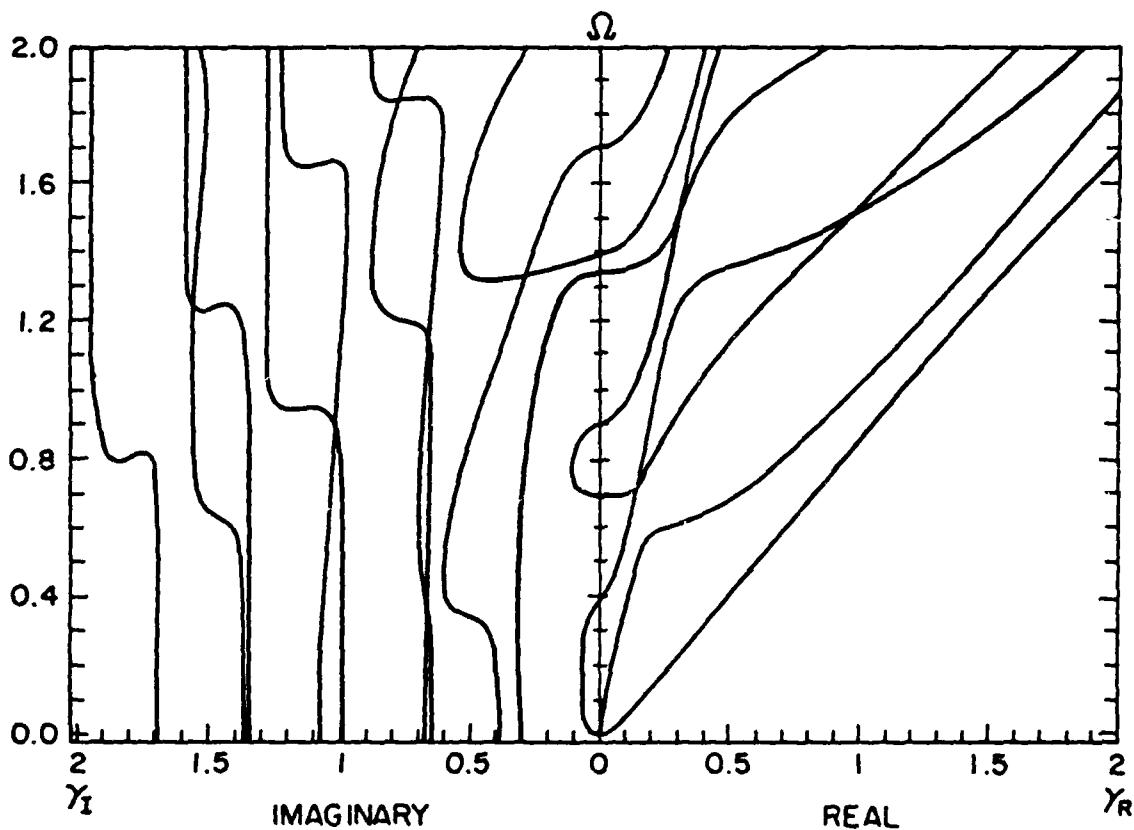


Fig. 3. Dispersion curves for $0^\circ/90^\circ/0^\circ$ plate. Here H is the total thickness of the plate, $\Omega = \frac{\omega H}{2\pi \sqrt{(c_{55}/\rho)_{\sigma}}}$, and $\gamma = \frac{kH}{2\pi}$.

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